

## Final exam of mathematics (calculus)

The problems are independent and sorted from least to most difficult. All answers should be justified (a correct answer with no justification will earn no point). Points associated to each problem are subject to change.

### Exercise 1 : Power of a complex number, 2 points

Let  $z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ . Compute  $z^{2019}$ .

### Exercise 2 : Limits, 3 points

Compute :

$$\lim_{x \rightarrow 0} \frac{\cos(x) - \frac{3}{2}\sin^2(x) - \cos(2x)}{x^3}$$

### Exercise 3 : Complex space, 3 points

Find all complex numbers  $z$  such that  $|z - 1 + i| = |z + 1 - i|$ . Express the set of such number as a geometric object of the complex space  $\mathbb{C}$

### Exercise 4 : Cooking pot, 4 points

Consider a cooking pot (viewed as an open cylinder with no lid) of fixed surface  $S_0$ , radius  $R$  and height  $h$ . What ratio  $h/R$  maximizes the volume of the cooking pot?

### Exercise 5 : Pension, 4 points

Nuria joins firm EN on her 22nd birthday. Her monthly wage debuts at 1200€ and increases each year (on her birthday) by 1.5% until she retires, 40 years later, on her 62nd birthday. On top of her wage, she earns a monthly bonus worth 10% her monthly wage.

In system  $A$ , Nuria's retirement pension is worth 75% of her last wage.

In system  $B$ , Nuria's retirement pension is worth 50% of her average revenue (wage + bonus) over her 40 years of work at EN.

What's the relative change (in percents) of Nuria's pension if there's a switch from system  $A$  to system  $B$ ?