Problem: find $m^* = \underset{|Am - e_i| \leq \gamma}{\operatorname{argmin}} \; m' A m$ with $\gamma > 0$

$e_i = (\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{n-i})$

$A \in S_n^+$ (psd).

$m \in \mathbb{R}_n$

Note $A = UDU^T$ the SVD of A with - U orthogonal ($U^T U = I_n$).

- D diagonal: $D = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$

$\lambda_1 \geq \ldots \geq \lambda_n = 0$.

Note r the rank of D (so that $+ \lambda_1, \ldots \lambda_r > 0$

$+ \lambda_{r+1}, \ldots \lambda_n = 0$.)

The problem is equivalent to finding

$$\tilde{m} = \underset{|Bm - e| \leq \gamma}{\operatorname{argmin}} \; m' D m \qquad \text{where } B = UD$$

And the solutions are related by $m^* = U\tilde{m}$

Initialization: start with a feasible m.

Update: - Loop over $j \in [\![1, n]\!]$ to update m_j as follows.

~~$B_j m_j = Bm - m_j B_j$ $\quad c = B_{-j} m_{-j} = Bm - m_j B_j$~~

~~- compute feasibility set $\mathcal{C}_j = \bigcap_{j'}$~~

$c = B_{-j} m_{-j} + B_j m_j$

note $b = B_j$, $e = e_i$

feasibility set for m_j for constraint j' is $|c_{j'} + m b_{j'} - e_{j'}| \leq \gamma$

And hence constraint set is $\mathcal{C} = \bigcap_{j'} \left[\frac{e_{j'} - c_{j'} - \gamma}{b_{j'}}, \frac{e_{j'} - c_{j'} + \gamma}{b_{j'}} \right]$

$$= \left[\max_{j'} \left(\frac{e_{j'} - c_{j'} - \gamma}{b_{j'}} \right), \min_{j'} \left(\frac{e_{j'} - c_{j'} + \gamma}{b_{j'}} \right) \right]$$

with the convention $\frac{x}{0} = \begin{cases} +\infty & \text{if } x \in \mathbb{R}_+^* \\ -\infty & \text{if } x \in \mathbb{R}_-^* \end{cases}$

the updated value of m_j is then $P_{\mathcal{C}}(0)$ (projection of 0 on the set $\mathcal{C}$, as 0 is the unconstrained solution.

- Iterate until convergence.